Reexamination of inflation in noncommutative space-time after Planck results

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An inflationary model in the framework of noncommutative space-time may generate a nontrivial running of the scalar spectral index, but usually induces a large tensor-to-scalar ratio simultaneously. With the latest observational data from the Planck mission, we reexamine the inflationary scenarios in a noncommutative space-time. We find that either the running of the spectral index is tiny compared with the recent observational result, or the tensor-to-scalar ratio is too large to allow a sufficient number of *e*-folds. As examples, we show that the chaotic and power-law inflation models with the noncommutative effects are not favored by the current Planck data.

PACS numbers: 98.80.Cq,98.80.Es

I. INTRODUCTION

Inflationary paradigm [1–3], which postulated an epoch of accelerated expansion in the very early Universe, has become one the most successful branches in modern cosmology. It not only eliminated a number of long-standing cosmological puzzles, such as the horizon, flatness, entropy, and monopole problems, but also provided a causal explanation of the primordial fluctuations. When the scales of the initial perturbations exceeded the Hubble radius, they lost physical correlations and got frozen. Later on, in the radiation- and matter-dominated eras, these inhomogeneities reentered the Hubble radius, seeding the large-scale structures in the Universe. The most important example is the anisotropies of the cosmic microwave background (CMB), which encoded the earliest and cleanest information from the cosmological inflation.

In the last decade, the observations of the CMB, especially from the Wilkinson Microwave Anisotropy Probe (WMAP) [4], have found enough evidence to support the predictions of the inflationary cosmology, which asserted that our Universe is spatially flat, with adiabatic and almost Gaussian primordial fluctuations, described by a nearly scale-invariant power spectrum. Very recently, the Planck mission released data, showing that the exact scale-invariant Harrison–Zel'dovich spectrum [5, 6] is ruled out at over 5σ confidence level (CL) by the Planck temperature data combined with the WMAP large-angle polarization (WP) data [7, 8]. The scalar spectral index measured by the Planck+WP data is $n_{\rm s} = 0.9603 \pm 0.0073$, which is very useful for constraining inflationary models; when the tensor component is included, the spectral index is not significantly changed, $n_{\rm s} = 0.9624 \pm 0.0075$. Besides, the Planck+WP data also set a tighter upper bound on the tensor-to-scalar ratio, $r < 0.12$ (at the pivot scale $k_* = 0.002\ {\rm Mpc}^{-1}$). It is shown in Ref. [8] that the space for the allowed standard inflationary models is shrunk (potentials with $V_{\phi\phi} < 0$ are preferred), and some models cannot provide a good fit to the Planck data.

The running of the scalar spectral index, ${\rm d}n_{\rm s}/{\rm d}\ln k$, is also measured by the Planck mission. It is found by the Planck+WP data that ${\rm d}n_{\rm s}/{\rm d}\ln k = -0.0134 \pm 0.0090$ (at the pivot scale $k_* = 0.05\ {\rm Mpc}^{-1}$), which is negative at 1.5σ level; at the same time, we have $n_{\rm s} = 0.9561 \pm 0.0080$ [8]. Though the evidence of a negative value for ${\rm d}n_{\rm s}/{\rm d}\ln k$ of $\mathcal{O}(10^{-2})$ is not statistically significant enough, such a result is still interesting for the physics of inflation, since the typical slow-roll inflationary models can only generate the running of $\mathcal{O}(10^{-3})$. Obviously, if the result of a large negative running of the spectral index is confirmed, a new window into physics of inflation will be opened. So it is very important to design inflationary models that predict a negative running of $\mathcal{O}(10^{-2})$ with an acceptable $n_{\rm s}$ and number of *e*-folds.

A way of realizing a large negative running is to consider a slow-roll inflation in a noncommutative space-time. Though it is still hard to construct a realistic noncommutative inflationary model, a toy model [9] has been extensively studied [10–19], and was especially examined with the WMAP 3-yr results [13–16]. Since there was a strong degeneracy between $n_{\rm s}$ and ${\rm d}n_{\rm s}/{\rm d}\ln k$ [20], the WMAP data favored a blue spectrum and a large negative running, with a large tensor-to-scalar ratio. It was shown that the WMAP results could be nicely explained by the noncommutative inflationary models [13–16]. However, the Planck mission now places much tighter constraints on the primordial power spectrum, preferring a red spectrum, with a negative running of $\mathcal{O}(10^{-2})$ and a small tensor-to-scalar ratio. Therefore, it is worthy of reexamining the status of the noncommutative inflationary models in light of the latest observational data from the Planck mission [7, 8]. This is the main purpose of the present paper.

This paper is organized as follows. First, in Sect. II, we briefly review the inflationary cosmology in noncommutative space-time, list the predictions for the primordial parameters, and discuss the possibility to realize a negative running of the spectral index in this framework. With these general preparations, in Sect. III, we move on to investigate in detail two specific inflationary scenarios, the chaotic and the power-law inflation models. We find that the noncommutative effects may give rise to a red power spectrum and a negative running in these models, but always induce a too large tensor-to-scalar ratio at the same time, so cannot thoroughly match the Planck data. Finally, we conclude in Sect. IV.

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II. INFLATION IN NONCOMMUTATIVE SPACE-TIME

A noncommutative space-time emerges as a natural consequence from string theory [21, 22]. For the noncommutative inflationary models, we refer the reader to Ref. [12] and the references therein. In any physical process, the noncommutation of time and space coordinate implies a new uncertainty relation,

$$\Delta t \Delta x \geq l_s^2 = \frac{1}{M_s^2},$$

where t and x are the physical time and space coordinate, l_s is the string length, which is the fundamental degree of freedom in string theory, and $M_s := l_s^{-1}$ is the string mass scale. In general, if M_s is close to the energy scale at which inflation took place, the noncommutative effects will accordingly impact the details of cosmological inflation. Consequently, observable imprints may be left on the late-time evolution of the Universe, e.g., the CMB angular power spectrum [9–11].

A. Noncommutative effects in inflation

In this paper, we restrict our discussion in the single-field slow-roll inflationary models, and explore the influences from the noncommutative effects on it. In this framework, the evolution of the homogeneous and isotropic background and the Klein–Gordon equation for the inflaton field will not be changed in noncommutative space-time,

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3m_P^2}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right],$$
$$\ddot{\phi} + 3H\dot{\phi} + V_\phi(\phi) = 0,$$

where $a(t)$ is the scale factor, $H := \dot{a}/a$ is the Hubble expansion rate, ϕ is the inflaton field, $V(\phi)$ is its potential (with the subscript $_\phi$ being the partial derivative with respect to ϕ), and $m_P := 1/\sqrt{8\pi G} = 2.44\times10^{18}$ GeV is the reduced Planck mass. For simplicity, we assume that the Universe is spatially flat. In the slow-roll approximation, the inflaton field slowly evolves down its potential, and three slow-roll parameters may thus be introduced as

$$\epsilon_V := \frac{m_P^2}{2}\left(\frac{V_\phi}{V}\right)^2, \quad \eta_V := m_P^2\frac{V_{\phi\phi}}{V}, \quad \xi_V := m_P^4\frac{V_\phi V_{\phi\phi\phi}}{V^2}.$$

Hence, the nearly exponential expansion of the Universe is achieved under the conditions $\epsilon_V \ll 1$ and $|\eta_V| \ll 1$.

Despite the unchanged evolution of the background space-time, the influences from the noncommutative effects show up in the evolutions of the cosmological perturbations and their power spectra. To express these more explicitly, a new time coordinate, namely, the modified conformal time $\tilde{\eta}$, is introduced. In this way, the equation of motion for the Fourier mode of the comoving curvature perturbation $\mathcal{R}_k$ reads

$$\mathcal{R}_k'' + 2\frac{z_k'}{z_k}\mathcal{R}_k' + k^2\mathcal{R}_k = 0, \tag{1}$$

where we denote the derivative with respect to $\tilde{\eta}$ by $'$, and

$$\frac{d\tilde{\eta}}{d\eta} = \left(\frac{\beta_k^-}{\beta_k^+}\right)^{1/2}, \quad z_k(\tilde{\eta}) = \frac{a\dot{\phi}}{H}\left(\beta_k^+\beta_k^-\right)^{1/4},$$
$$\beta_k^+ = \frac{1}{2}\left[a^2(\eta + l_s^2k) + a^2(\eta - l_s^2k)\right],$$
$$\beta_k^- = \frac{1}{2}\left[\frac{1}{a^2(\eta + l_s^2k)} + \frac{1}{a^2(\eta - l_s^2k)}\right],$$

where η is the conformal time. The deviation of the equation of motion for $\mathcal{R}_k$ in noncommutative space-time from that in the ordinarily commutative space-time is encoded in the terms β_k^+ and β_k^-, and furthermore can be characterized by a noncommutative parameter μ [12],

$$\mu := \left(\frac{Hkl_s}{a}\right)^2 = \left(\frac{Hk}{aM_s}\right)^2.$$

Solving Eq. (1), we arrive at several basic parameters for the power spectrum of the comoving curvature perturbation (for detailed derivations of the following results, we refer the reader to Ref. [12]):

1. the power spectrum,

$$\mathcal{P}_\mathcal{R} = \frac{k^3}{2\pi^2}|\mathcal{R}_k|^2 = \frac{V}{24\pi^2 m_P^4\epsilon_V}(1+\mu)^{2\eta_V - 6\epsilon_V - 4}, \tag{2}$$

2. the scalar power spectral index,

$$s = n_s - 1 := \frac{d\ln\mathcal{P}_\mathcal{R}}{d\ln k} = 2\eta_V - 6\epsilon_V + 16\epsilon_V\mu, \tag{3}$$

3. the running of the scalar spectral index,

$$\alpha_s := \frac{dn_s}{d\ln k} = -24\epsilon_V^2 + 16\epsilon_V\eta_V - 2\xi_V - 32\epsilon_V\eta_V\mu, \tag{4}$$

4. the tensor-to-scalar ratio,

$$r = 16\epsilon_V. \tag{5}$$

From Eqs. (2)–(5), we may clearly observe the noncommutative effects, parameterized by μ, on the usual results of the inflationary cosmology. In the limit of vanishing noncommutative effects, $\mu \to 0$, all these results automatically reduce to the usual expressions in the standard slow-roll inflationary paradigm.

B. Running of the spectral index in noncommutative inflation

Now we take the running of the scalar spectral index α_s as an example to give a very general discussion of the noncommutative effects on the primordial parameters. From Eqs. (3) and (5), we have $\eta_V = \frac{1}{2}(\frac{3}{8}r + s - r\mu)$. Therefore, α_s can be reexpressed as

$$\alpha_s = r^2\mu^2 - \left(\frac{7}{8}r + s\right)r\mu + \frac{3}{32}\left(r + \frac{16}{3}s\right)r - 2\xi_V.$$

Usually, it is not easy to construct a large ξ_V in the known inflationary models, so in the following we neglect ξ_V for simplicity. Thus,

$$\alpha_s = r^2\mu^2 - \left(\frac{7}{8}r + s\right)r\mu + \frac{3}{32}\left(r + \frac{16}{3}s\right)r. \tag{6}$$

Equation (6) indicates that α_s is a quadratic function of the noncommutative parameter μ and is thus possible to be negative, as it is straightforward to see that the discriminant for Eq. (6) is positive definite,

$$\Delta = r^2\left[\frac{3}{8}r^2 + \left(\frac{1}{8}r - s\right)^2\right] > 0.$$

We classify α_s into three cases:

1. For $s > 0$, α_s is negative, if $\mu_- < \mu < \mu_+$, with $\mu_\pm = \frac{1}{2r^2}\left[r\left(\frac{7}{8}r + s\right) \pm \sqrt{\Delta}\right]$.
2. For $s < 0$ and $r < -\frac{16}{3}s$, α_s is negative, if $0 < \mu < \mu_+$.
3. For $s < 0$ and $r > -\frac{16}{3}s$, α_s is negative, if $\mu_- < \mu < \mu_+$.

From these analyses, we can conclude that there exists the possibility of a negative running of the spectral index in the noncommutative inflationary models. This is consistent with the discussions in Ref. [13]. We should admit that α_s may also be negative in the inflationary models in commutative space-time. As can be seen from Eq. (6), $\alpha_s = \frac{3}{32}(r + \frac{16}{3}s)r$, when $\mu = 0$. Therefore, α_s is negative, if $s < 0$ and $r < -\frac{16}{3}s$. But with a new parameter μ from the noncommutative effects, α_s may be negative for much larger ranges of r and s.

In Refs. [13–16], the noncommutative inflationary models were compared to the WMAP 3-yr results. However, at that time, the measurements of the spectral index n_s and its running α_s were still imprecise, $n_s = 1.21^{+0.13}_{-0.16}$ and $\alpha_s = -0.102^{+0.050}_{-0.043}$ [20]. The upper bound for the tensor-to-scalar ratio r was even looser, $r < 1.5$ at the 95% CL (WMAP data only) [20]. We see that a blue spectrum with a large negative running was favored, and large tensor components were consistent with these data. In Ref. [13], it was shown that the noncommutative chaotic inflation models could realize a considerable running index for the ϕ^2 and ϕ^4 cases, but a low number of e-folds, say, $N \sim 14$, is required. Subsequently, it was pointed out in Ref. [14] that the noncommutative chaotic inflation models could provide a large negative running of the spectral index within a reasonable range of the number of e-folds, provided that the power n in the potential $V(\phi) \sim \phi^n$ was enhanced roughly to $n \sim 12$–18. In addition, for the Kachru–Kallosh–Linde–Maldacena–McAllister–Trivedi brane inflation model [23], the noncommutative effects not only could nicely explain the large negative running of the spectral index, but also significantly relaxed the fine-tuning for the parameter β [15, 16] (see also Refs. [24–26] for the fine-tuning problem).

It has been shown by the Planck mission that compared to previous experiments, the Planck data make the space of possible inflationary models narrower. In particular, a preference for a negative running of n_s at modest statistical significance is still held, which has been challenging the design of slow-roll inflationary models. Thus, it is of interest to re-scrutinize the noncommutative inflationary models with the current Planck data. We wish to know if the noncommutative mechanism still works when confronting the Planck data. At least, we should make certain whether the noncommutative inflationary models help to provide a better fit to the data or make the situation worse.

In what follows, we quote the fit results for the Λ cold dark matter (ΛCDM) model, with a two-parameter extension, i.e., the ΛCDM+r+α_s model, from the Planck data combined with the WMAP large-scale polarization data and the baryon acoustic oscillation (BAO) data (henceforth Planck+WP+BAO) [8]:

$$\begin{aligned} n_s &= 0.9607 \pm 0.0063 \quad (68\%\ \text{CL}), \\ \alpha_s &= -0.021^{+0.012}_{-0.010} \quad (68\%\ \text{CL}), \\ r &< 0.25 \quad (95\%\ \text{CL}), \end{aligned} \tag{7}$$

which are given at the pivot scale $k_* = 0.05\ \text{Mpc}^{-1}$. Comparing to the WMAP results [20], we find that the spectral index n_s in this case is less than 1 rather than greater than 1 (red, not blue spectrum). This is not a good signal for the noncommutative effects, as from Eq. (3) we see that the noncommutative effects always make the power spectrum more blue, since ϵ_V must be positive. Also, we find that in this case a negative running of the spectral index is favored at about 2σ level. From Eq. (4), we observe that, if η_V is positive, the noncommutative effects will help to realize a negative running; otherwise, the role of the noncommutative effects is to prevent a negative running. This is a bad hint for the noncommutative inflation, because the Planck data prefer potentials with $V_{\phi\phi} < 0$, i.e., $\eta_V < 0$. The constraint on the tensor-to-scalar ratio is relaxed compared to the case with no running, due to an anti-correlation between r and α_s in the Planck+WP+BAO data. But such an upper bound for r is still fairly strict for constraining noncommutative inflation.

We will show in the following that the noncommutative effects may induce a negative running of the spectral index α_s, but will simultaneously generate a too large tensor-to-scalar ratio r. In fact, as shown immediately in Sect. III, the balance between α_s and r can never be achieved in the usual inflationary models in noncommutative space-time. As a result, the noncommutative inflationary models cannot survive under the strict constraints in Eq. (7), and are thus excluded by the current data.

III. NONCOMMUTATIVE EFFECTS IN SPECIFIC INFLATIONARY SCENARIOS

In this section, some most frequently discussed inflationary scenarios are inspected and reexamined in detail in noncommutative space-time. We find that the noncommutative inflationary models are in serious conflict with the current Planck+WP+BAO data.

A. Chaotic inflation scenario

The chaotic inflation scenario [27] is a typical large-field model, in which the potential takes the form $V(\phi) \sim \phi^n$, and the inflaton field ϕ is usually displaced from the minimum of $V(\phi)$ by an amount of the order of $m_{\rm P}$. The calculations of the slow-roll parameters in this model are a standard procedure in inflationary cosmology,

$$\epsilon_V = \frac{n}{4N}, \quad \eta_V = \frac{n-1}{2N}, \quad \xi_V = \frac{(n-1)(n-2)}{4N^2},$$

where N is the number of e-folds of inflation. Furthermore, the spectral index, the running of the spectral index, and the tensor-to-scalar ratio in noncommutative space-time are [12],

$$s = n_{\rm s} - 1 = \left(\mu - \frac{n+2}{8n}\right) r, \tag{8}$$

$$\alpha_{\rm s} = -\frac{n-1}{4n}\left[\mu + \frac{n+2}{8n(n-1)}\right] r^2, \tag{9}$$

$$r = \frac{4n}{N}. \tag{10}$$

From Eqs. (8) and (9), we get a red spectral index, if $\mu < \frac{n+2}{8n}$, and a negative definite running of the spectral index, $\alpha_{\rm s} < -\frac{n+2}{32n} r^2$. Therefore, in some sense, the noncommutative effects do help to induce a larger negative running of the spectral index (compared to that in the commutative case), but further analyses will reveal that these noncommutative effects also lead to a too large tensor-to-scalar ratio unavoidably.

From Eqs. (8) and (9), we can also solve r and μ as the functions of s and $\alpha_{\rm s}$,

$$r = \frac{4(n-1)}{n+2}\left[\sqrt{s^2 - \frac{2n(n+2)}{(n-1)^2}\alpha_{\rm s}} - s\right], \tag{11}$$

$$\mu = \frac{n+2}{8n} - \frac{(n-1)s}{8n\alpha_{\rm s}}\left[\sqrt{s^2 - \frac{2n(n+2)}{(n-1)^2}\alpha_{\rm s}} + s\right]. \tag{12}$$

With the Planck+WP+BAO constraints [8],

$$s = n_{\rm s} - 1 = -0.0393 \pm 0.0063, \quad \alpha_{\rm s} = -0.021^{+0.012}_{-0.010}, \tag{13}$$

it is not difficult to attain the allowed ranges for r and μ. Moreover, from Eqs. (8)–(10), it is straightforward to figure out the r–$n_{\rm s}$ and $\alpha_{\rm s}$–$n_{\rm s}$ relationships,

$$r = \frac{n_{\rm s} - 1}{\mu - \frac{n+2}{8n}},$$

$$\alpha_{\rm s} = -\frac{n+2}{32n} r^2 - \frac{n-1}{4n}(n_{\rm s} - 1) r = -\frac{n(n+2)}{2N^2} - \frac{n-1}{N}(n_{\rm s} - 1).$$

Now we pick two most often used cases, the ϕ^2 and ϕ^4 inflation models, as examples.

1. ϕ^2 inflation model

In this case, $n = 2$. From Eqs. (8)–(12), we have

$$s = \left(\mu - \frac{1}{4}\right) r, \quad \alpha_{\rm s} = -\frac{1}{8}\left(\mu + \frac{1}{4}\right) r^2, \quad r = \frac{8}{N}, \tag{14}$$

and

$$r = \sqrt{s^2 - 16\alpha_{\rm s}} - s,$$

$$\mu = \frac{1}{4} - \frac{s}{16\alpha_{\rm s}}\left(\sqrt{s^2 - 16\alpha_{\rm s}} + s\right).$$

The expected region of the tensor-to-scalar ratio r and the noncommutative parameter μ in the ϕ^2 inflation model is shown in Fig. 1. It is clear to see that the noncommutative ϕ^2 inflation model gives rise to a too large tensor-to-scalar ratio, $r > 0.41$, seriously inconsistent with the Planck+WP+BAO result, $r < 0.25$ [8]. In fact, in Fig. 1, we simply treat s and $\alpha_{\rm s}$ as free parameters, with the constraints in Eq. (13), and neglect their correlation, while this is already enough for us to conclude that the noncommutative effects are not favored by the current observations, because the allowed ranges of r and μ will be even narrower, with the correlation of s and $\alpha_{\rm s}$ taken into account.

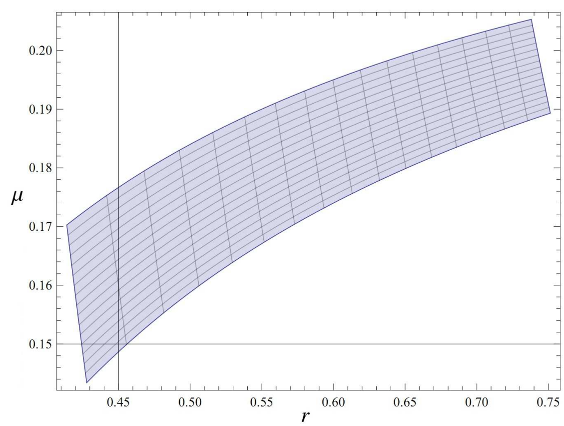

FIG. 1: The expected region of the tensor-to-scalar ratio r and the noncommutative parameter μ in the ϕ^2 inflation model under the consideration of the 1σ ranges of $n_{\rm s}$ and $\alpha_{\rm s}$ in Eq. (7). We find that the noncommutative effects lead to a too large tensor-to-scalar ratio, $0.41 < r < 0.75$. Here we simply treat s and $\alpha_{\rm s}$ as free parameters, and the lower bound of r will be even larger, if the correlation of s and $\alpha_{\rm s}$ is considered.

In order to avoid the problem of a too large tensor-to-scalar ratio, we find from Eq. (14) that the number of e-folds must be large enough, $N > 8/0.25 = 32$. But this causes another problem. From Eq. (14), we have

$$\alpha_{\rm s} = -\frac{4}{N^2} - \frac{n_{\rm s} - 1}{N}. \tag{15}$$

We find that the running of the spectral index $\alpha_{\rm s}$ is a linear function of the spectral index $n_{\rm s}$. However, for a large enough

N, the slope of the function will be too small to produce a negative enough running. Let us choose $N = 40$ ($r = 0.2$) as an example to compare with the Planck+WP+BAO results in the α_s–n_s plane [8]. Our result is shown in Fig. 2. We observe that $\alpha_s = -0.0005$ at $n_s = 0.92$ and $\alpha_s = -0.0025$ at $n_s = 1$. This means that with a suitable number of e-folds, although the noncommutative effects may generate a negative running of the spectral index α_s, this running is always too small. A larger N will make the situation worse, as the slope in Eq. (15) becomes even smaller. To see how serious this trouble is, we pick the central values of α_s and n_s from the Planck+WP+BAO data [8], $\alpha_s = -0.021$ and $n_s = 0.9607$, and by solving Eq. (15), we get $N = 17.1$, which is far from enough for a reasonable inflationary model. The above analysis demonstrates that the noncommutative effects in the ϕ^2 inflation model cannot provide useful help in explaining the large negative running of the spectral index. From Fig. 2, we see that the noncommutative ϕ^2 inflation model (with $r = 0.2$) lies outside of the 1σ region of Planck+WP+BAO constraint on the ΛCDM+α_s+r model (red contours).

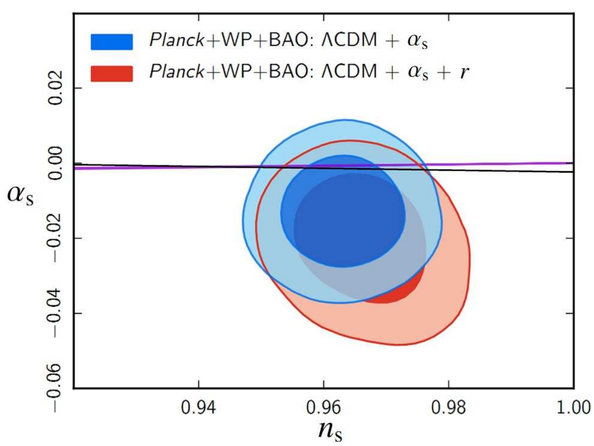

FIG. 2: The predicted relationship of the running of the spectral index α_s and the spectral index n_s (black line) in the ϕ^2 inflation model in noncommutative space-time, with the number of e-folds chosen to be $N = 40$. Contours (68% and 95% CLs) in the α_s–n_s plane are from the joint Planck+WP+BAO constraints; here we use a copy of Fig. 2 in Ref. [8]. For comparison, we keep the purple strip that shows the prediction for usual chaotic inflation models in commutative space-time with $50 < N < 60$. On the contrary to this strip, the noncommutative effects lead to a negative slope for the α_s–n_s relation. We find a rather small α_s in the noncommutative model, $\alpha_s = -0.0005$ at $n_s = 0.92$ and $\alpha_s = -0.0025$ at $n_s = 1$. For the central value of the spectral index $n_s = 0.9607$ [8], we get $\alpha_s = -0.001518$. These values are tiny compared with the central value $\alpha_s = -0.021$ [8], indicating that the noncommutative ϕ^2 inflation model is not favored by the current Planck+WP+BAO observations at the 68% CL (red contours). Larger number of e-folds makes things worse, as the slope of the function in Eq. (15) is even smaller.

Furthermore, for the r–n_s relationship, from Eq. (14), we have

$$r = \frac{n_s - 1}{\mu - \frac{1}{4}}.$$

For a positive r, $\mu < 0.25$. We show the theoretical predictions of the model in Fig. 3, in which we choose $\mu = 0.05$, 0.10, and 0.15, respectively, and compare these three cases with the ordinary ϕ^2 inflation model in commutative space-time. Since the noncommutative ϕ^2 inflation model also predicts a negligible running, given a reasonable number of e-folds, say, 50–60, the theoretical predictions (the green, red, and blue lines) should be compared to the blue contours in the r–n_s plane. We see that the noncommutative ϕ^2 inflation model with $\mu \gtrsim 0.05$ is ruled out at the 95% CL. Even if the comparison is made to the red contours, the model with $\mu \gtrsim 0.05$ is ruled out at the 68% CL, and with $\mu \gtrsim 0.15$ is ruled out at the 95% CL.

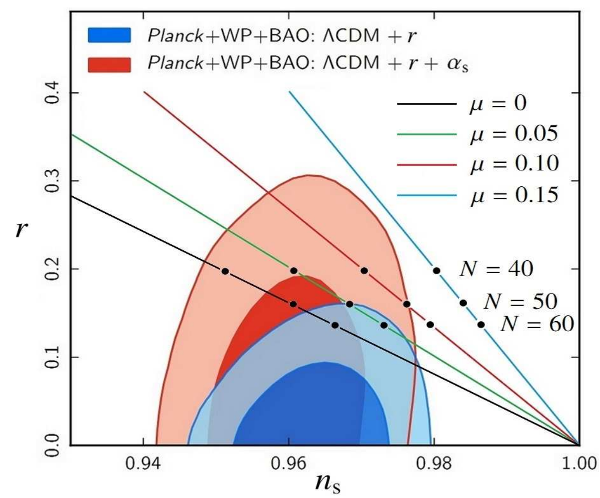

FIG. 3: The predicted relationship of the tensor-to-scalar ratio r and the spectral index n_s in the ϕ^2 inflation model in noncommutative space-time. Contours (68% and 95% CLs) in the r–n_s plane are from the joint Planck+WP+BAO constraints; here we use a copy of Fig. 4 in Ref. [8]. The black line indicates the ϕ^2 inflation model in commutative space-time, and the green, red, and blue lines show the r–n_s relationships, with the noncommutative parameter μ taken to be 0.05, 0.10, and 0.15, respectively. We find that the larger μ is, the more inconsistent the noncommutative models are with the Planck+WP+BAO data. If r is lower than its upper bound $r < 0.25$ [8], a large enough number of e-folds N is needed. Here we take $N = 40$, 50, and 60, respectively. Since the noncommutative ϕ^2 inflation model predicts a negligible running, given a reasonable number of e-folds, the theoretical predictions (the green, red, and blue lines) should be compared to the blue contours in the r–n_s plane. We see that the noncommutative ϕ^2 inflation model with $\mu \gtrsim 0.05$ is ruled out at the 95% CL. Even if the comparison is made to the red contours, the model with $\mu \gtrsim 0.05$ is ruled out at the 68% CL, and with $\mu \gtrsim 0.15$ is ruled out at the 95% CL.

In a word, we conclude that the ϕ^2 chaotic inflation model in noncommutative space-time is not favored by the current Planck+WP+BAO results [8]. If we want to explain the results of n_s and α_s in Eq. (7), a large value of r will be inevitable, which severely exceeds the current observational upper bound. If we abandon explaining the large running, we will find that the noncommutative effects do not provide a better fit to the data, but make the situation even worse; in this case, the noncommutative ϕ^2 inflation model with $\mu \gtrsim 0.05$ is

ruled out by the current data at the 95% CL.

2. ϕ^4 inflation model

In this case, $n = 4$. From Eqs. (8)–(12), we have

$$s = \left(\mu - \frac{3}{16}\right) r, \quad \alpha_s = -\frac{3}{16}\left(\mu + \frac{1}{16}\right) r^2, \quad r = \frac{16}{N},$$

and

$$r = 2\left(\sqrt{s^2 - \frac{16\alpha_s}{3}} - s\right),$$
$$\mu = \frac{3}{16} - \frac{3s}{32\alpha_s}\left(\sqrt{s^2 - \frac{16\alpha_s}{3}} + s\right).$$

In fact, the ordinary ϕ^4 inflation model in commutative space-time has been ruled out at more than 3σ level by the current Planck data [8]. Now let us see the status of the noncommutative case. Similarly as the ϕ^2 inflation model, if the values of n_s and α_s in Eq. (7) are taken, the expected region of the tensor-to-scalar ratio r and the noncommutative parameter μ in the ϕ^4 inflation model is shown in Fig. 4. In this case, the lower bound of r is even higher, $r > 0.51$, in obvious contradiction with its current observational bound. Thus, for the ϕ^4 model, the noncommutative effects give even worse predictions than those in the ϕ^2 inflation model.

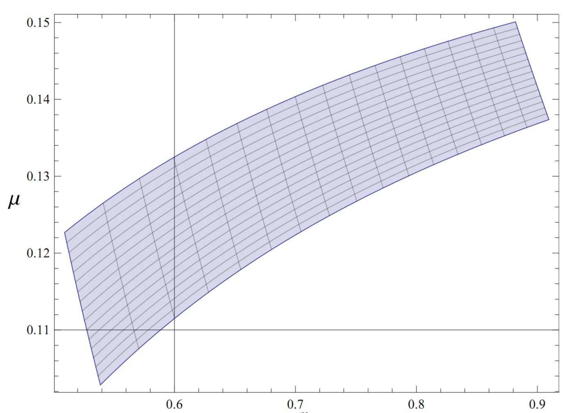

FIG. 4: The expected region of the tensor-to-scalar ratio r and the noncommutative parameter μ in the ϕ^4 inflation model under the consideration of the 1σ ranges of n_s and α_s in Eq. (7). The noncommutative effects lead to a even larger tensor-to-scalar ratio, $0.51 < r < 0.91$. This means that the noncommutative ϕ^4 inflation model is more inconsistent with the current observational data.

The considerations on the α_s–n_s and r–n_s relationships in the noncommutative ϕ^4 inflation model are totally analogous to those in the ϕ^2 inflation model, only with more severe discrepancies, so we do not duplicate our analyses.

B. Power-law inflation scenario

The power-law inflation scenario [28] is driven by a potential of the type $V(\phi) = \lambda^4 \exp\left(-\sqrt{\frac{2}{p}}\frac{\phi}{m_P}\right)$. Here p is some number, characterizing the expansion rate, and λ describes the energy scale of inflation. The corresponding solution of the scale factor in this scenario is exactly in the power-law form, $a(t) = \left[\frac{t}{(p+1)l}\right]^p$, due to which this scenario bears its name (λ is re-parameterized as l in $a(t)$).

In this model, an additional exit mechanism is needed, since the model itself cannot stop inflation. Usually, we assume that such a mechanism exists, and the cosmological perturbations are not affected by this mechanism. The ordinary power-law inflation model in commutative space-time has been ruled out at about 3σ level by the current Planck data [8]. Now we explore the noncommutative case.

The power spectrum, the scalar spectral index, the running of the spectral index, and the tensor-to-scalar ratio were calculated in detail in Ref. [12],

$$\mathcal{P}_{\mathcal{R}} = Ak^{-\frac{2}{p-1}}\left[1 - \sigma\left(\frac{k_c}{k}\right)^{\frac{4}{p-1}}\right], \tag{16}$$

$$s = n_s - 1 = -\frac{2}{p-1} + \frac{4\sigma}{p-1}\left(\frac{k_c}{k}\right)^{\frac{4}{p-1}}, \tag{17}$$

$$\alpha_s = -\frac{16\sigma}{(p-1)^2}\left(\frac{k_c}{k}\right)^{\frac{4}{p-1}}, \tag{18}$$

$$r = \frac{16}{p}, \tag{19}$$

where

$$A = \left[\frac{(2p-1)p}{(p+1)^2 l^2}\right]^{\frac{p}{p-1}} \frac{p l_P^2}{8\pi^2}, \tag{20}$$

$$k_c = \left[\frac{(2p-1)p}{(p+1)^2}\right]^{\frac{p+1}{4}} \frac{1}{l_s}\left(\frac{l_s}{l}\right)^p, \tag{21}$$

$$\sigma = \frac{4(p-2)(2p+1)p^2}{(p-1)(2p-1)(p+1)^2}, \tag{22}$$

with $l_P = m_P^{-1} = 8.10 \times 10^{-35}$ m $= 2.62 \times 10^{-57}$ Mpc being the Planck scale. From Eqs. (16)–(18), we find that $\mathcal{P}_{\mathcal{R}}$, s, and α_s are the functions of three parameters, A, k_c, and σ, and further of another three parameters, p, l, and l_s. Thus, with the joint Planck+WP+BAO results [8] for $\mathcal{P}_{\mathcal{R}}$, s, and α_s, we are able to constrain the allowed ranges of all these parameters.

From Eqs. (17) and (18), we get $s = -\frac{2}{p-1} - \frac{p-1}{4}\alpha_s$, so

$$p = 1 - \frac{2}{\alpha_s}\left(\sqrt{s^2 - 2\alpha_s} + s\right).$$

For their central values in Eq. (13), $s = -0.0393$ and $\alpha_s = -0.021$, we obtain

$$p = 17.1.$$

As a consequence, from Eq. (19), we have

$$r = 0.936.$$

Hence, we again find an over large tensor-to-scalar ratio in the power-law inflation model in noncommutative space-time. In other words, we say that the scale factor grows not fast enough as needed in this model, since its power should be $p > 16/0.25 = 64 \gg 17.1$, if $r < 0.25$ [8].

Below we give all the other relevant parameters, σ, k_c, A, l, l_s, and M_s. For $p = 17.1$, from Eq. (22), we find

$$\sigma = 3.55.$$

Thus, from Eq. (18), we have

$$k_c = 3.98 \times 10^{-6}\ \mathrm{Mpc}^{-1},$$

where we pick the central value $\alpha_s = -0.021$ and the pivot scale $k_* = 0.05\ \mathrm{Mpc}^{-1}$ [8]. Moreover, from Eq. (16), we obtain

$$A = 2.29 \times 10^{-9}\ \mathrm{Mpc}^{-0.124},$$

where $\mathcal{P}_{\mathcal{R}}(k_* = 0.05\ \mathrm{Mpc}^{-1}) = 2.196 \times 10^{-9}$ [7]. Furthermore, from Eq. (20), we get

$$l = 1.23 \times 10^{-27}\ \mathrm{m}.$$

Finally, from Eq. (21), we arrive at

$$\begin{aligned} l_s &= 4.15 \times 10^{-31}\ \mathrm{m} = 5.13 \times 10^3 l_P, \\ M_s &= 4.76 \times 10^{14}\ \mathrm{GeV} = 1.95 \times 10^{-4} m_P. \end{aligned}$$

We should state that all the values above are taken as the central values, extracted from the joint Planck+WP+BAO data [7, 8]. It is unnecessary to discuss the specific meaning of each parameter, since it is only the trouble of the large tensor-to-scalar ratio that is already enough for us to discard the noncommutative power-law inflation model.

IV. DISCUSSION AND SUMMARY

In this paper, we reexamine the inflationary models in noncommutative space-time, with the constraints from the latest observational data from the Planck mission [7, 8]. Firstly, we demonstrate in general that the noncommutative effects can help to induce nontrivial modifications to the primordial parameters. Then, as examples, we explore the possible noncommutative effects in two specific noncommutative inflation scenarios, the chaotic and power-law inflation models, but find incurable discrepancies between the predictions from these models and the joint Planck+WP+BAO data. First, if the observational constraints of the spectral index and the running of the spectral index, $n_s = 0.9607 \pm 0.0063$ and $\alpha_s = -0.021^{+0.012}_{-0.010}$ (68% CL) [8], are satisfied, the noncommutative effects will always produce a too large tensor-to-scalar ratio: $r > 0.41$ for the ϕ^2 inflation model, $r > 0.51$ for the ϕ^4 inflation model, and $r = 0.936$ (in this case we only take the best-fit values as an example) for the power-law inflation. Furthermore, if we let the observational upper bound for the tensor-to-scalar ratio, $r < 0.25$ [8], be satisfied, we find that the running of the spectral index α_s will be tiny, still around $O(10^{-3})$, i.e., under such circumstances the noncommutative effects cannot provide considerable help in generating large running spectral index. For instance, the maximum of α_s is -0.0025 in the noncommutative ϕ^2 inflation model, which is much smaller than its central value -0.021 from the joint Planck+WP+BAO data [8]. Last, we find that, even though we give up explaining the large running spectral index, the noncommutative effects do not provide a better fit to the data but make the situation worse. For example, considering the r–n_s relationship, we find that the noncommutative effects give rise to a larger tensor-to-scalar ratio, if the spectral index is fixed; as a result, the ϕ^2 inflation model with the noncommutative parameter $\mu \gtrsim 0.05$ is ruled out at the 95% CL by the joint Planck+WP+BAO data. Altogether, we can conclude for safety that the inflationary models in noncommutative space-time are disfavored by the current Planck data.

Acknowledgments

We are very grateful to Yun-He Li and Dominik J. Schwarz for helpful discussions. NL is supported by the National Natural Science Foundation of China (Grant No. 11105026). XZ is supported by the National Natural Science Foundation of China (Grant Nos. 10705041, 10975032, and 11175042) and by the National Ministry of Education of China (Grant Nos. NCET-09-0276 and N100505001).

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